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Sensitivity of Goodness-of-Fit Indices to Lack of Measurement Invariance with Categorical Indicators and Many Groups

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Research questions

- How sensitive are standard SEM goodness-of-fit indices to lack of measurement invariance with categorical data and large second-level sample sizes (10-50 groups)?
- How critical, in the same context, are different levels of non-invariance (inevitable in large samples) with respect to substantive inferences, e.g. latent means comparison?
- Related work: Rutkovski and Svetina 2014, 2017; Svetina and Rutkovski 2017; Kim et al. 2017; Pokropek, Davidov and Schmidt 2019.

Study design

- Fit indices: CFI, TLI, RMSEA, SRMR/WRMR
- Model: one factor, four items (each with four response categories).
- Number of groups: $\{10, 30, 50\}$.
- Amount of non-invariance: 9 conditions (full invariance - two with scalar non-invariance - six with scalar/metric non-invariance).
- Other model misspecifications: No/one non-zero residual covariance/two non-zero residual covariances.
- In sum: 81 conditions, 500 replications for each
- Missing values: 10% observations in each group (MCAR)
- Group sizes: 30% $-$ 1000, 40% $-$ 1500, 30% $-$ 2000.

Parameter values 1

Latent means:

$\mathbb{N}(0; 1)$

Latent variances:

$\mathbb{U}(0.6; 1.4)$

Factor loadings:

1. $\{0.75, 0.75, 0.6, 0.6\}$ in all groups
2. **1st** and **2nd**: trunc. $\mathbb{N}(0.75, 0.05; L = 0.6, U = 0.9)$
   3rd and **4th**: trunc. $\mathbb{N}(0.6, 0.05; L = 0.45, U = 0.75)$
3. **1st** and **2nd**: trunc. $\mathbb{N}(0.75, 0.05; L = 0.6, U = 0.9)$
   3rd: trunc. $\mathbb{N}(0.6, 0.05; L = 0.45, U = 0.75)$
   4th: $\mathbb{U}(\sqrt{0.1}, 0.75)$
4. **1st**: trunc. $\mathbb{N}(0.75, 0.05; L = 0.6, U = 0.9)$
   2nd: $\mathbb{U}(sqrt0.1, 0.9)$
   3rd: trunc. $\mathbb{N}(0.6, 0.05; L = 0.45, U = 0.75)$
   4th: $\mathbb{U}(\sqrt{0.1}, 0.75)$

Parameter values 2

Thresholds:

1. **1st**: $\{-0.8, 0, 0.8\}$
   2nd: $\{-0.8, 0, 0.8\}$
   3rd: $\{-0.6, 0, 0.6\}$
   4th: $\{-0.6, 0, 0.6\}$
2. **All**: trunc. $\mathbb{N}(\tau_{jc}^{Cond1}, 0.05; L = \tau_{jc}^{Cond1} - 0.2, U = \tau_{jc}^{Cond1} + 0.2)$
3. **All**: trunc. $\mathbb{N}(\tau_{jc}^{Cond1}, 0.2; L = \tau_{jc}^{Cond1} - 0.35, U = \tau_{jc}^{Cond1} + 0.35)$

where τ_{jc}^{Cond1} is the threshold value for the j-th item and the c-th response category in Condition 1.

Parameter values 2

Total item variances:

$\mathbb{U}(0.8, 1.2)$

Residual variances:

Item i's total variance minus its squared loading in the fully invariant condition (0.75 or 0.6)

Residual covariances (Item 1 ~~ Item 2 and Item 3 ~~ Item 4):

1. $\{0, 0\}$ in all groups
2. **1st**: zero in all groups
   2nd: trunc. $\mathbb{N}(0.1, 0.1, L = 0, U = 0.2)$
3. **1st**: trunc. $\mathbb{N}(0.05, 0.1, L = -0.1, U = 0)$
   2nd: trunc. $\mathbb{N}(0.1, 0.1; L = 0, U = 0.2)$

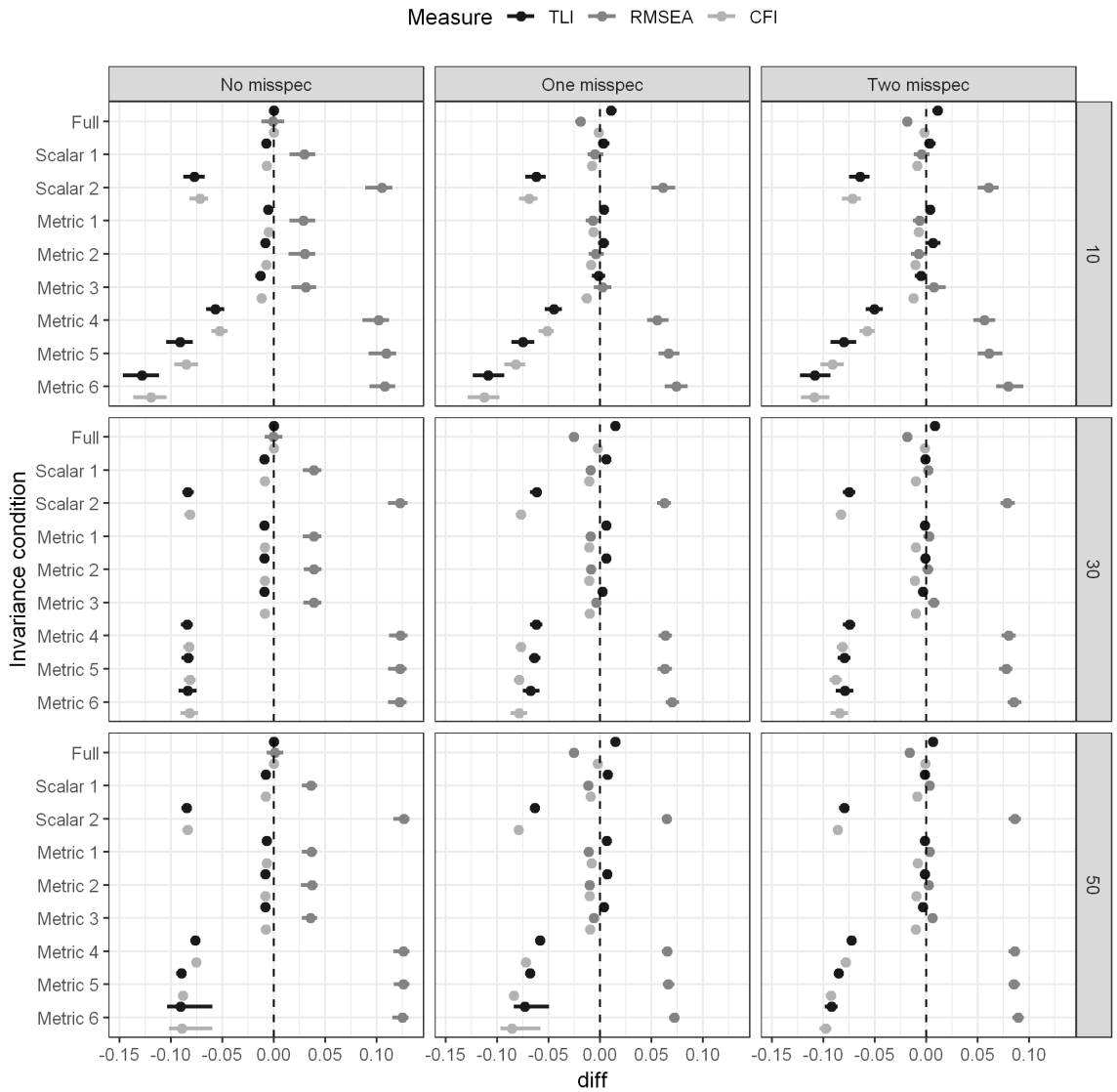
Invariance conditions

1. Full Inv.: Loadings 1 + Thresholds 1
2. Scalar 1: Loadings 1 + Thresholds 2
3. Scalar 2: Loadings 1 + Thresholds 3
4. Metric 1: Loadings 2 + Thresholds 2
5. Metric 2: Loadings 3 + Thresholds 2
6. Metric 3: Loadings 4 + Thresholds 2
7. Metric 4: Loadings 2 + Thresholds 3
8. Metric 5: Loadings 3 + Thresholds 3
9. Metric 6: Loadings 4 + Thresholds 3

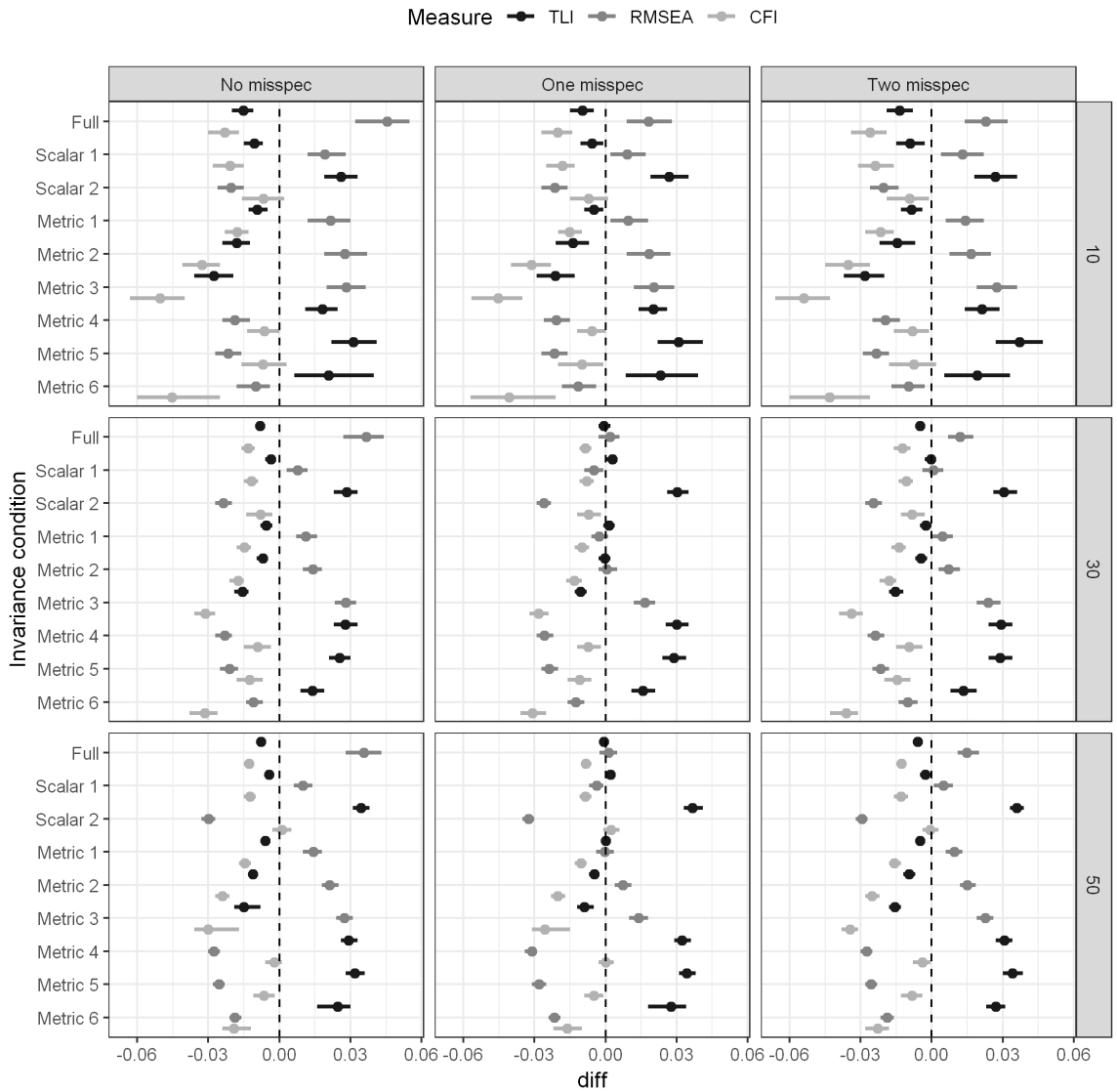
Simulation and Estimation

- Simulation: R packages `simsem` and `lavaan`
- Estimation: MPLUS 7.11 (via the R package `MplusAutomation`)
- Estimation methods:
  - MLR
  - WLSMV (MPLUS default identification)
  - WLSMV (Wu and Estabrook's identification approach)

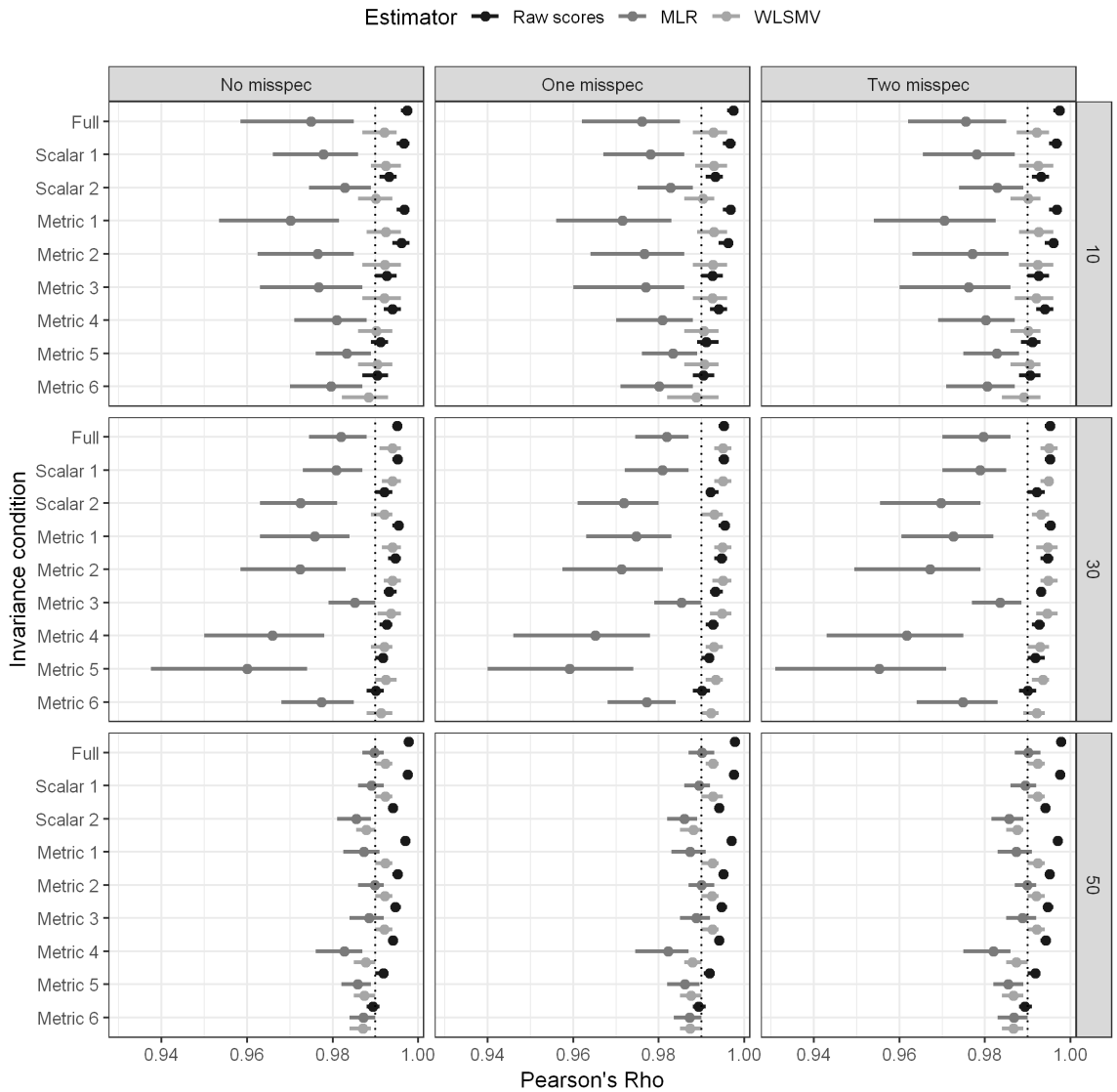
Configural vs. Threshold (Wu and Estabrook)



Threshold vs. Threshold + Loading (Wu and Estabrook)



True vs. Estimated Means Correlations



Results

- CFI seems to be the “best-performing” fit index; SRMR is the second-best (but only with MLR estimation)
- Other misspecifications negatively (and non-linearly) affects both the absolute and the relative model fit for all fit indices and invariance levels
- Second-level sample size negatively affects sample variability of fit indices but has little impact on their average values.
- Loading and intercept/threshold non-invariances generally have a multiplicative effect on model fit
- All fit indices often fail to discriminate between approximately invariant data and fully invariant data.
- It is difficult to propose universally applicable cutoff values; *ad hoc* simulations should guide researchers’ decisions.
- Even [relatively] highly non-invariant models may produce reliable (*comparable*?) latent means estimates (??)

Proposed cutoff values (MLR and WLSMV-1 estimation)

- Configural invariance:
  - **MLR**: CFI > 0.985; SRMR < 0.02
  - **WLSMV**: CFI > 0.985
- Loading invariance:
  - **MLR**: ΔCFI > $-$ 0.01; ΔSRMR < 0.01
  - **WLSMV**: ΔCFI > $-$ 0.005
- Intercept/Threshold invariance:
  - **MLR**: ΔCFI > $-$ 0.01; ΔSRMR < 0.01; ΔTLI > $-$ 0.005; ΔRMSEA > 0.005
  - **WLSMV**: ΔCFI > $-$ 0.005; ΔTLI > 0.00; ΔRMSEA < 0.00

Critical values above are based on the (approximate) average values of the 2.5th (CFI and TLI) or 97.5th (SRMR and RMSEA) percentiles of the respective fit indices averaged across all conditions in which full invariance of a given level holds

Proposed cutoff values (WLSMV-2: Wu&Estabrook)

- Configural invariance:
  - CFI > 0.99
- Threshold Invariance:
  - ΔCFI > $-$ 0.005(0.002); ΔCFI > 0.00; ΔRMSEA < 0.00
- Threshold + Loading Invariance:
  - ΔCFI > $-$ 0.02

Critical values above are based on the (approximate) average values of the 2.5th percentiles of the CFI averaged across all conditions in which full invariance of a given level holds

Thank you very much for your attention!

Please send your questions, comments and feedback at bssokolov@gmail.com

References

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