Can Missing Patterns in Covariates Improve Imputation for Missing Data?

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Background

Most (survey) data sets have missing data:

Treatment:

- Imputation of plausible values to receive a data set without missings (e.g., sequential imputation step)

Problems:

- Bias due to oversimplified models in the sequential imputation step
- Information that “respondent did not answer a question” is lost
- If missing data mechanism is Missing Not At Random (MNAR), the item missing pattern can be informative for the imputed values

Basic Idea

- Include "Missing" as own category in imputation model to improve imputation accuracy and therefore estimators from survey data
- Tree-based methods (e.g., random forest) can incorporate this additional information and account for complex interactions, (Doove, Van Buuren, and Dusseldorp 2014)
- Increasing efficiency by skipping sequential imputation steps

$\Rightarrow$ Easy to implement in current software

Previous Research

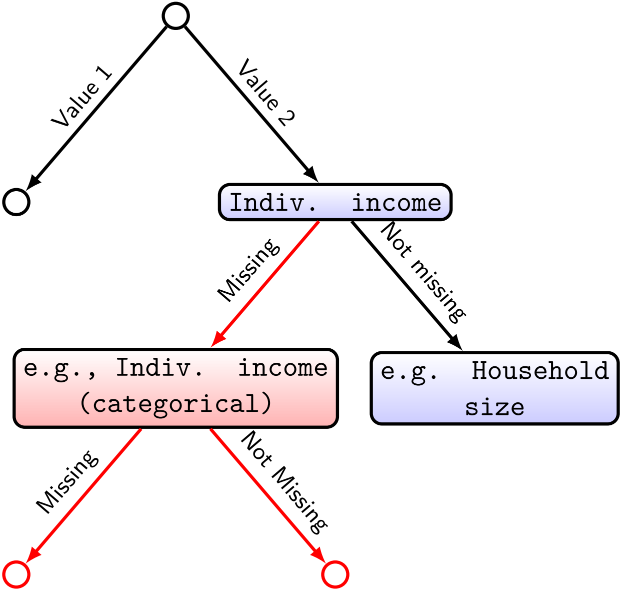
- Loh et al. (2018) use missing values in a regression and classification tree (GUIDE) to impute missing values
- Ding and Simonoff (2010) show that random forest can handle incomplete covariates by coding "missing" as its own category

Potentially New Approach

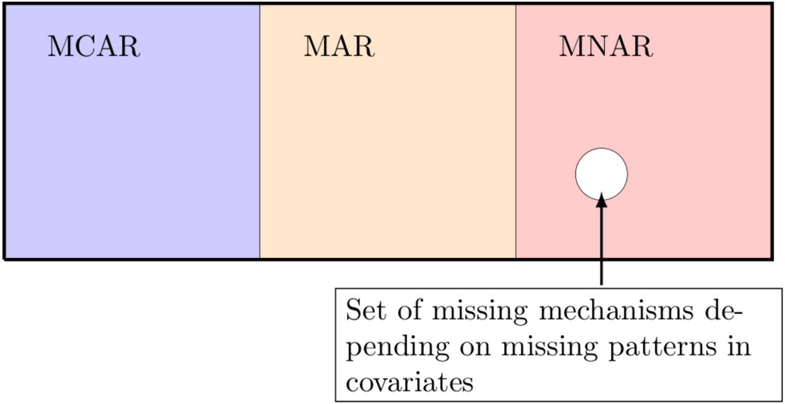
- Combine both approaches, "tree-based imputation" and "missing as its own category" with CART and random forest:
  - Categorical covariates: Treat "Missing" as its own category
  - Continuous covariates: Code "Missing" to arbitrary value "(far) away" from the actual data

⇒ One kind of pattern mixture model which partition data by patterns of missing values (Little 1993)

Example: Potential Part of Tree for Imputing Household Income



Assumption on Missing Data - Venn Diagram:



Theoretical Considerations

- Little (2019 - forthcomming):
  MAR (standard) vs. specific MNAR assumption in Loh et al (2018)
- Many covariates with missing values: potential advantage due to more available data
- Reasonable when missing due to "not applicable" cases

Simulation Study - Set Up (1)

- Generating covariables: $X_i \sim N(0,1)$ and $Z_i \sim N(0,1)$
- Generating response indicators for X and Z:

$$R_{Z,i} \sim Ber(p_Z) \text{ and } R_{Z,i} = \begin{cases} 1 \text{ for } p_Z \geq u_{Z,i}, \\ 0 \text{ for } p_Z < u_{Z,i} \end{cases}$$

where $u_{Z,i} \sim Unif(0,1)$

Simulation Study - Set Up (2)

- Generating outcome variable Y:
  $Y_i = \beta_0 + \beta_1 X_i + \beta_2 Z_i + \beta_3 R_{Z,i} + \beta_4 X_i R_{Z,i} + \epsilon_i$
  where $\epsilon_i \sim N(0,1)$
- Generating response indicator for Y:
  $P(R_{Y,i} = 1) = logit^{-1}(p_Y + \delta_1 X_i + \delta_2 Z_i + \delta_3 R_{Z,i} + \delta_4 Y_i)$

with p_Y as the baseline response rate,

$$\text{and } R_{Y,i} = \begin{cases} 1 \text{ for } P(R_{Y,i} = 1) \geq u_{Y,i}, \\ 0 \text{ for } P(R_{Y,i} = 1) < u_{Y,i} \end{cases}$$

where $u_{Y,i} \sim Unif(0,1)$

$$\Rightarrow Y_{\text{obs},i} = \begin{cases} Y_i \text{ if } R_{Y,i} = 1, \\ missing \text{ if } R_{Y,i} = 0 \end{cases} \qquad Z_{\text{obs},i} = \begin{cases} Z_i \text{ if } R_{Z,i} = 1, \\ missing \text{ if } R_{Z,i} = 0 \end{cases}$$

Simulation Study - Data Structure

Table 1: Resulting data structure

Y_{obs}	X_{obs}	Z_{obs}
Y	X	Z
Y	X	miss
miss	X	Z
miss	X	miss

Simulation Study - Procedure

- Single imputation with linear models, CART and random forest using $X_{obs,i}, Y_{obs,i}, Z_{obs,i}$
- Additionally, CART and random forest using "Missing" information as own category
- Assessment on RMSE of regression coefficiences after imputation in a substantive model - Here:

$$Y_{imp,i} \sim X_{obs,i} + Z_{imp,i}$$

Simulation - Parameters

Table 2: Implemented parameter values

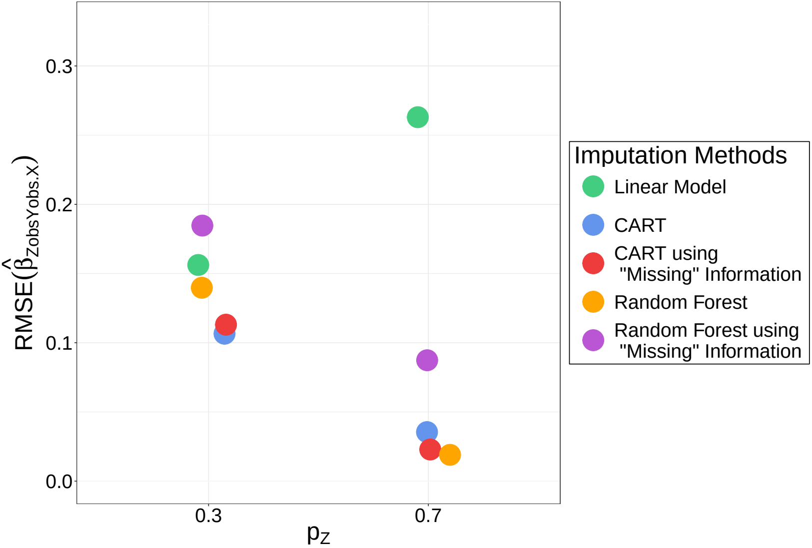
Y	Parameter	β_1	β_2	β_3	β_4
	Values	1	1	$\{0; 2\}$	$\{0; 2\}$
Response propenstiy	Parameter	δ_1	δ_2	δ_3	δ_4
	Values	1	$\{0; 1\}$	$\{0; 1\}$	$\{0; 1\}$
Baseline response	Parameter	p_Y	p_Z		
	Values	0.5	$\{0.3; 0.7\}$		

$\Rightarrow$ MAR situation if $\beta_4 = \delta_4 = 0$

$\Rightarrow$ MNAR situation if $\beta_4 \neq 0 \vee \delta_4 \neq 0$

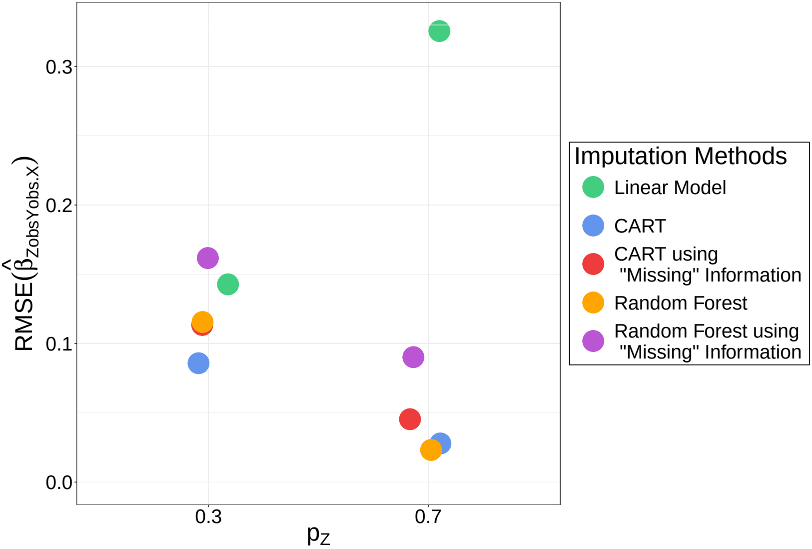
Simulation Results - MAR

Parameter values: $\beta_3 = 2$, $\beta_4 = 0$ and $\delta_2 = \delta_4 = 0$, $\delta_3 = 1$



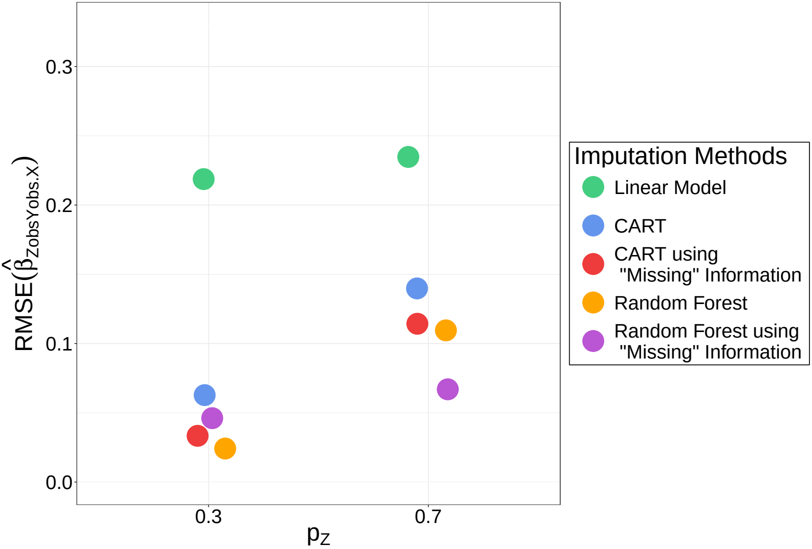
Simulation Results - MNAR

Parameter values: $\beta_3 = \beta_4 = 2$ and $\delta_2 = \delta_4 = 0$, $\delta_3 = 1$



Simulation Results - MNAR

Parameter values: $\beta_3 = \beta_4 = 2$ and $\delta_2 = \delta_3 = \delta_4 = 1$



Future Research

1. Simulation extension for non-normal distributed variables (e.g., binary variables)
2. Evaluation on survey data linked to administrative records
3. Accounting for imputation uncertainty in variance estimation

Thank you for your attention!

Any questions?

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